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Evolution of the Order Parameter after Bubble Collisions

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If a first-order phase transition is terminated by collisions of new-phase bubbles, there will exist a period of nonequilibrium between the time bubbles collide and the time thermal equilibrium is established. We study the behavior of the order parameter during this phase. We find that large nonthermal fluctuations at this stage tend to restore symmetry, i.e., the order parameter is smaller than its eventual thermal equilibrium value. We comment on possible consequences for electroweak baryogenesis.

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It has long been known that symmetry may be restored at high temperature in local thermodynamic equilibrium (LTE) [1]. Recently it was realized that certain nonequilibrium (NEQ) conditions can be even more efficient for symmetry restoration [2]. An example of such a nonequilibrium state can arise naturally after inflation in the so-called preheating era [3, 4]. In fact, symmetry may be restored in the NEQ state even if it is not restored in the LTE state formed by thermalization of the NEQ state. Detailed numerical studies [5] confirm that fluctuations of inflaton decay products is large enough for symmetry restoration, as well as for several other important effects, including baryogenesis [6], and supersymmetry breaking [7], and generation of a background of relic gravitational waves [8].

States with properties similar to those in preheating, namely, anomalously large fluctuations and highly NEQ conditions, can arise in other situations as well. It was suggested in Ref. [9] that if bubble collisions produce large numbers of soft scalar particles carrying quantum numbers associated with a spontaneously broken symmetry, the phenomenon of (or tendency toward) symmetry restoration may occur. The basic point is that bubble collisions create NEQ conditions with a large number of "soft" quanta of average energy smaller than the equivalent LTE temperature corresponding to instantaneous conversion of the bubble energy density into radiation. Since it may require several scattering times for the low-energy quanta to form a thermal distribution, it is rather reasonable to consider the NEQ period as a separate epoch. This is generally referred to as the 'preheating' epoch in a manner similar to the preheating phase of slow-roll inflation [3].

The tendency of symmetry restoration in NEQ conditions after bubble collisions may be readily understood by the following (somewhat oversimplified) reasoning. Let us imagine that particles χ are produced in the bubble wall collisions and are charged under some symmetry group, so that their mass, m_χ, depends upon some scalar field ϕ (the order

parameter of the symmetry) as $m_\chi^2(\phi) = m_0^2 + g\phi^2$.[1] Here, g represents a combination of numerical factors and a coupling constant. As a simple example we might assume that the ϕ-dependent mass originates from a potential term of the form $V_{\chi\phi} = (1/2)g\phi^2\chi^2$. As opposed to the large-angle scattering processes required for thermalization, forward-scattering processes do not alter the distribution function of the particles, but simply modify the dispersion relation. This is true in NEQ conditions, as well as the familiar LTE conditions. Forward scattering is manifest, for example, as ensemble and scalar background corrections to the particle masses. Since the forward-scattering rate is usually larger than the large-angle scattering rate responsible for establishing a thermal distribution, the nonequilibrium ensemble and scalar background corrections are present before the initial distribution function relaxes to its thermal value. These considerations allow us to impose the dispersion relation $\omega^2 = \mathbf{p}^2 + m_\chi^2(\phi)$ for NEQ conditions.

The leading contribution of the particles created by bubble collisions to the one-loop effective potential of the scalar field ϕ can be shown to be $\Delta V(\phi) \simeq (n/E)m_\chi^2(\phi)$ [2, 10], where n and E are the number density and the energy of the χ quanta, respectively. We may write the potential for the NEQ configuration as $\Delta V(\phi) = B_{\rm NEQ}\phi^2$, where $B_{\rm NEQ} = gn/E$. In NEQ conditions, the coefficient $B_{\rm NEQ}$ may be quite large, indeed larger that the corresponding equilibrium coefficient which scales like $T_{\rm RH}^2$, $T_{\rm RH}$ defined as the temperature of the universe when the thermal spectrum of radiation is first obtained. Therefore, the tendency of symmetry restoration may turn out to be rather independent of $T_{\rm RH}$. We also notice that since the energy E scales like the inverse of the bubble wall width Δ, $E \sim \Delta^{-1}$, one can suggest that the effect of soft particles on symmetry restoration is stronger for thick bubble walls.

The aim of the present paper is to investigate numerically the issue of symmetry

[1] Of course χ particles may coincide with the ϕ particles themselves, but in this example the colliding bubbles are **not** made from the field ϕ. Otherwise there can be some effect, but the original symmetry will not be restored.

restoration in bubble wall collision. We will explicitly show that at the final stage of first-order phase transitions when bubble collisions occur, nonthermal quanta are produced, and that they tend to restore symmetry. This tendency can be quantified as a shift of the order parameter ϕ from its equilibrium value toward smaller values. We will also confirm the conjecture about the dependence of the strength of symmetry restoration upon the bubble wall width. Finally, we will comment on the possible implications that our result may have for electroweak baryogenesis.

Let us concentrate on a theory with a single scalar field ϕ (the χ particles of the above discussions must be identified with the ϕ) with Lagrangian density

$$\mathcal{L} = \frac{1}{2}\partial\phi_\mu\partial\phi^\mu - \frac{1}{2}m^2\phi^2 + \frac{1}{3}c\phi^3 - \frac{1}{4}\lambda\phi^4 - V_0, \tag{1}$$

where V_0 is a constant. We introduce the dimensionless variables $\varphi \equiv \phi/\phi_0$, $\tau \equiv \sqrt{\lambda}\phi_0\, t$, and $\xi = \sqrt{\lambda}\phi_0\, x$, where ϕ_0 will be fixed later. In the new variables the factor $\lambda\phi_0^4$ is an overall multiplication factor for the Lagrangian ($\widetilde{m} = m/\sqrt{\lambda}\phi_0$, $\widetilde{c} = c/\lambda\phi_0$, $\widetilde{V_0} = V_0/\lambda\phi_0^4$)

$$\mathcal{L} = \lambda\phi_0^4\left[\frac{1}{2}\partial\varphi_\mu\partial\varphi^\mu - \frac{1}{2}\widetilde{m}^2\varphi^2 + \frac{1}{3}\widetilde{c}\varphi^3 - \frac{1}{4}\varphi^4 - \widetilde{V_0}\right] \equiv \lambda\phi_0^4\left[\frac{1}{2}\partial\varphi_\mu\partial\varphi^\mu - V(\varphi)\right] . \tag{2}$$

The overall factor will not enter the equation of motion. The final step is a choice of a potential, which we choose such that $dV/d\varphi = \varphi\,(\varphi-1)\,(\varphi-\varphi_m)$. The equation of motion is then

$$\Box\varphi + \varphi\,(\varphi-1)\,(\varphi-\varphi_m) = 0 \; . \tag{3}$$

With this choice of $dV/d\phi$ the extrema of the potential are transparent: it has minima at $\varphi = 0$ and $\varphi = 1$ and a local maximum at $\varphi = \varphi_m$ (we thus fix the parameter φ_m to be in the range $0 < \varphi_m < 1$). We shall assume $\varphi = 1$ corresponds to the true vacuum, i.e., $V(0) > V(1)$. Making the connection with Eq. (1), we conclude that $\phi = \phi_0$ is the field strength in the true vacuum, and the constants entering Eq. (1) are $m^2 = \varphi_m\,\lambda\phi_0^2$ and $c = (1+\varphi_m)\,\lambda\phi_0$. We shall require the absence of cosmological constant in the true

vacuum, $V(1) = 0$; this gives $V_0 = (1 - 2\varphi_m)\lambda\phi_0^4/12$. Since we consider the true vacuum to be at $\varphi = 1$ and the false vacuum at $\varphi = 0$, we can further restrict the parameter φ_m to be in the range $0 < \varphi_m < 0.5$.

Note that only one parameter, φ_m, enters the equation of motion in the rescaled variables. This is a key point. The evolution of any initial field configuration, $\varphi(\tau = 0, \xi)$, for fixed φ_m will be the same in the rescaled variables, regardless of the coupling constant λ.

The initial field configuration for the problem at hand corresponds to a set of new-phase critical bubbles expanding in the false vacuum. Note that the evolution of a critical bubble is also defined by Eq. (3), and consequently it is fixed when φ_m is fixed. However, the bubble nucleation probability is a more complicated function of the other variables (note that nucleation became unsuppressed when $\varphi_m \to 0$, i.e, when the potential barrier disappears). The nucleation probability will determine the initial separation of critical bubbles (in space, as well as in time). In our numerical integration we will consider the mean separation of bubble nucleation sites as another free parameter of the model . Fixing it gives one extra constraint on the set of parameters λ, ϕ_0 and φ_m.

After nucleation, new phase bubbles expand and collide. After collisions, the spatial distribution of the magnitude of φ resembles a random superposition of many wavelength modes—a configuration with large field fluctuations. It is important that the system is classical and can be described by Eq. (3) from the time of bubble nucleation, through the time of bubble collisions and the condition of large field fluctuations.

The random-wave configuration is quickly established after bubble collisions; essentially it is established on the time-scale of bubble collisions since there is no small parameters in Eq. (3). Eventually the waves interact and LTE is established. Since transforming the NEQ distribution function into an LTE distribution function involves producing states with small occupation number, the coupling constant λ will enter the time scale

for the establishment of LTE. This time scale can be very long if λ is small, so the NEQ configuration can exist for a long time. This phase has specific properties which are the subject of our study here.

First, let us recall what is expected in the final LTE state. The LTE temperature can be found using energy conservation

$$g_* \frac{\pi^2}{30} T_{\rm LTE}^4 = V_0 = \lambda \phi_0^4 \frac{(1-2\varphi_m)}{12} \,, \tag{4}$$

which gives

$$T_{\rm LTE} = \left(\frac{\lambda}{g_*}\right)^{1/4} \phi_0 \left[\frac{5(1-2\varphi_m)}{\pi^2}\right]^{1/4} \equiv \lambda^{1/4} \phi_0 \, b, \tag{5}$$

where b is a constant of order unity and $g_*(T)$ is the number of relativistic degrees of freedom at temperature T. Note that $T_{\rm LTE}$ approaches zero as λ approaches 0. Due to interactions with the medium, LTE values of the model parameters, e.g., the effective mass, are different than vacuum values. The value of the parameters can be calculated as loop corrections to the action. Most important is the change of the effective mass, $m_{\rm eff}^2(T) = m^2 + \lambda T^2/4$. At very high temperatures $m_{\rm eff}^2(T)$ becomes positive, even if the zero-temperature value of m^2 was negative. This is a signal that broken symmetries are restored at high temperatures [1].

In the model of Eq. (1) which we consider here, the symmetry can not be restored again after bubble collisions, but the temperature dependent contribution to the effective mass will be nonzero. Using Eq. (5) we find that it scales with coupling constant as $\lambda^{3/2}$, and tends to zero as λ tends to 0. Note for what follows that the temperature-dependent correction to the mass can be written in more general form, as $m_{\rm eff}^2 = m^2 + 3\lambda\langle\phi^2 - \langle\phi\rangle^2\rangle$.

Let us now find the mean value of the field φ in thermal equilibrium with temperature given by Eq. (5). To leading order in the coupling constants, the equation $dV_{\rm eff}/d\varphi = 0$ becomes

$$(\varphi_m + 3\sqrt{\lambda} b^2)\varphi - (1+\varphi_m)\varphi^2 + \varphi^3 - (1+\varphi_m)\sqrt{\lambda} b^2/12 = 0 \,, \tag{6}$$

where terms proportional to $\sqrt{\lambda}$ are reminiscent of temperature-dependent corrections to the effective potential rewritten in terms of our dimensionless variables. We see that the solution of this equation tends to $\varphi = 1$ when $\lambda \to 0$. In other words, the mean value of the field φ in thermal equilibrium (established after the phase transition is completed) differs very little from the vacuum expectation value if the coupling constant is small.

We can study the process of bubble collisions and subsequent chaotization by numerically integrating Eq. (3). We defined a 3-dimensional box of size l on a grid of size 128^3 employing periodic boundary conditions. With periodic boundary conditions every bubble in the box is mirrored by its (infinitely repeating) reflections. As the bubble expands to fill the box, it will collide with its reflections, and there is no need to put more than one bubble inside the box to study bubble collisions. So we have restricted ourselves to an initial configuration corresponding to just one critical bubble of the true phase in the box. In this case the size of the box, l, corresponds to the mean initial separation of bubbles in units of $1/\lambda^{1/2}\phi_0$. We integrated the equation of motion for $l = 4, 8, 10$ and 12, corresponding to progressively larger bubbles at collision time. We used $\varphi_m = 0.1$ for the only parameter in the equation of motion.

The results for the time dependence of zero mode of the field, $\varphi_0 = \langle\varphi\rangle$, is presented in Fig. 1, where $\langle\ldots\rangle$ means the spatial average (over grid points). We see that after bubbles have collided ($\tau > 16$ for $l = 4$ and $\tau > 40$ for $l = 8$), the zero mode does not relax to is vacuum value, $\varphi_0 = 1$, but oscillates near some smaller value. We define $\varphi_0 \equiv \overline{\langle\varphi\rangle}$, where bar denotes the time average over several oscillations. We find $\varphi_0 \approx 0.93$ in the case $l = 8$ and $\varphi_0 \approx 0.87$ with $l = 4$ at $\tau \sim 80$. Note that φ_0 rises slightly with τ, which is the sign of ongoing relaxation. We do not present results for $l = 10$ and $l = 12$ since they do not differ appreciably from the case $l = 8$ (φ_0 at $l = 12$ is larger by an about 0.01 than the corresponding value for $l = 8$).

The deviation of φ_0 from the vacuum value is not unexpected since a random field

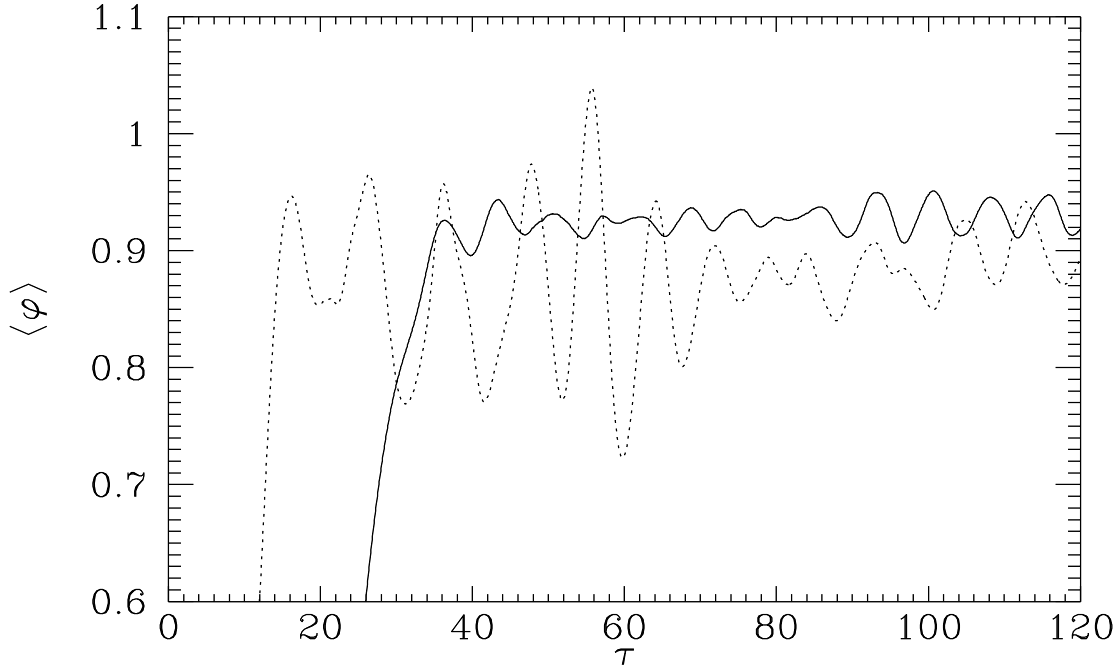

Figure 1: Time dependence of the zero-momentum mode, $\varphi_0 = \langle\varphi\rangle$. The dotted line corresponds to initial bubble separation of $l = 4$, the solid line corresponds to $l = 8$.

of classical waves is created after bubble collision, i.e., $\mathrm{Var}(\varphi) \equiv \langle\varphi^2\rangle - \langle\varphi\rangle^2$, is nonzero. The time dependence of the variance is shown in the Fig. 2. Note again that with fixed initial conditions the variance does not depend upon λ, i.e., it has a nonthermal origin.

At $\tau \sim 80$, with $l = 8$ we have $\mathrm{Var}(\varphi) \approx 0.036$ and with $l = 4$ we find $\mathrm{Var}(\varphi) \approx 0.08$. Again we employ time averaging over several oscillations. At small λ those values are much larger than its equivalent LTE value $\mathrm{Var}(\varphi) = T_{\mathrm{LTE}}^2/12$ (see Eq. (5)). The fact that $\mathrm{Var}(\varphi)$ in NEQ can exceed its equivalent LTE value by many orders of magnitude was the main point of Ref. [2] which studied the preheating phase after inflation, and of Ref. [9] which studied conditions following bubble collisions. Our work supports the claim in Ref. [9] that NEQ phase transitions can occur in models which contain more degrees of freedom than the simple toy model of Eq. (1).

Let us see whether we can understand the deviation of the zero mode from its vacuum

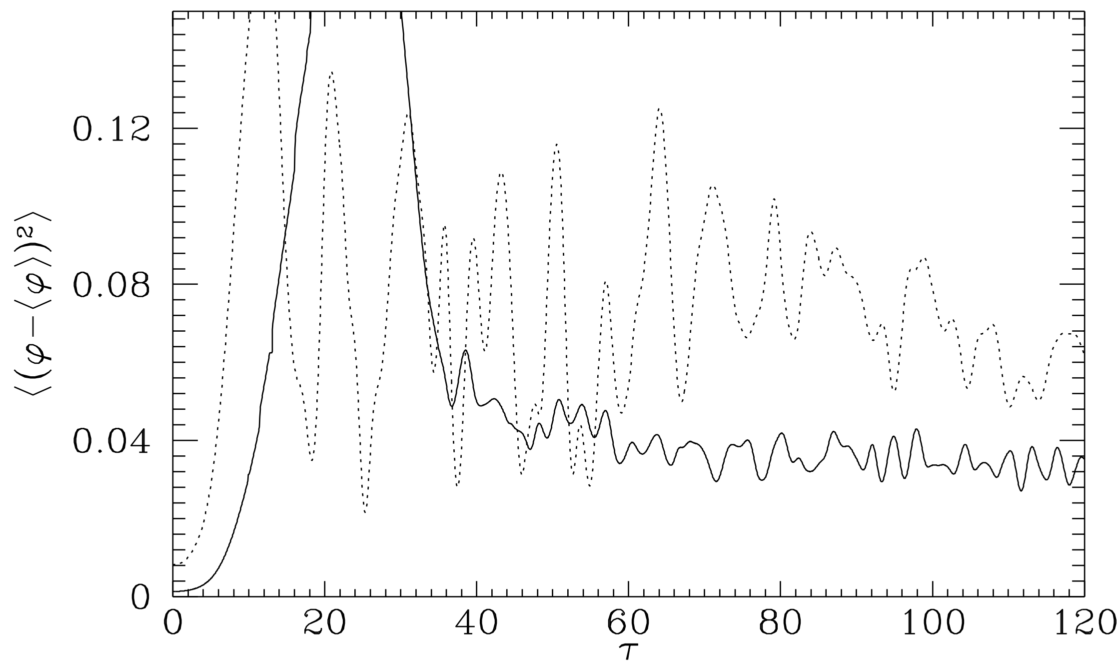

Figure 2: Time dependence of the variance, $\langle\varphi^2\rangle - \langle\varphi\rangle^2$. The dotted line corresponds to initial bubble separation of $l = 4$, the solid line corresponds to $l = 8$..

value by the existence of a nonzero $\mathrm{Var}(\varphi)$. Let us decompose the field as $\varphi = \varphi_0 + \delta\varphi$, and substitute this decomposition into the equation $dV/d\varphi = 0$. We find in the Hartree approximation

$$(\varphi_m + 3\langle\delta\varphi^2\rangle)\varphi_0 - (1+\varphi_m)\varphi_0^2 + \varphi_0^3 - (1+\varphi_m)\langle\delta\varphi^2\rangle = 0\,. \tag{7}$$

Assuming in addition the deviation of φ_0 from 1 to be small, we find

$$\varphi_0 = 1 - \frac{2-\varphi_m}{1-\varphi_m+3\langle\delta\varphi^2\rangle}\langle\delta\varphi^2\rangle\,. \tag{8}$$

Using $\varphi_m = 0.1$ and the values of $\langle\delta\varphi^2\rangle$ inferred from Fig. 2, we find $\varphi_0 = 0.93$ for $l = 8$ and $\varphi_0 = 0.87$ for $l = 4$, which are in excellent agreement with the results presented in Fig. 1.

We can also understand the dependence upon l, the initial bubble separation. The larger the initial bubble separation, the longer bubbles will expand before they collide.

As a bubble expands, its wall thickness decreases. Hence, colliding bubbles in the $l = 8$ calculation are thinner than in the $l = 4$ case. Following the discussion in Ref. [9], we expect the average energy of the quanta created in wall collisions to scale as Δ^{-1}, where Δ is the wall thickness at collision. Since the effect of the background on the effective potential scales as $n/E \propto \Delta$, we expect the $l = 4$ calculation to result in a larger departure from the vacuum value. This expectation is confirmed by the results shown in the figures.

Even though we only examine a particularly simple model, we conjecture that a deviation of φ_0 from its thermal equilibrium value in the aftermath of bubble collisions may have important consequences for some applications of first-order phase transitions, e.g., electroweak baryogenesis. In any scenario where the baryon asymmetry is generated during a first-order electroweak phase transition, the asymmetry is generated in the vicinity of bubble walls, and a strong constraint on the ratio between the vacuum expectation value of the Higgs field inside the bubble and the temperature must be imposed, $\langle\phi(T)\rangle/T > 1$ [11]. This bound is necessary for the just created baryon asymmetry to survive the anomalous baryon violating interactions inside the bubble, and may be translated into an severe upper bound on the physical mass of the scalar Higgs particle. Combining this bound with the LEP constraint already rules out the possibility of electroweak baryogenesis in the standard model of electroweak interactions, and even leaves little room for electroweak baryogenesis in the minimal supersymmetric extension of the standard model [12]. Since the rate of anomalous baryon number violating processes scales like $\exp(-\langle\phi\rangle/T)$, it is clear that even a small change in the vacuum expectation value of the Higgs scalar field from its equilibrium value may be crucial for electroweak baryogenesis considerations. Our results suggest that imposing the bound $\langle\phi(T)_{\rm EQ}\rangle/T > 1$ may not be a sufficient condition for successful electroweak baryogenesis. NEQ effects at the completion of the phase transition may reduce the expectation

value of the Higgs field, thus enhancing the anomalous baryon number violating rate with respect to its equilibrium value, making the upper bound on the Higgs mass more severe. Applications of our considerations to the electroweak transition may result in a fatal blow to many scenarios involving extensions of the standard model where the baryon asymmetry is generated during the electroweak phase transitions.

The model we consider in this paper is quite simple, but it illustrates several points. The most important result is that NEQ conditions following bubble collisions can have a dramatic effect upon the effective potential. Although the model we study is too simple to result in symmetry restoration, the numerical results confirms the assumptions made in Ref. [9] about the efficiency of NEQ conditions. We mentioned a possible direct application of our results to electroweak baryogenesis, but we believe that the phenomenon of NEQ effects will have other implications as well.

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